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Symmetries in Nature

Klaus Mainzer*

Dedicated to Professor Hans Primas on the occasion of his 60th birthday

Symmetry, dissymmetry, chirality etc. are well-known topics in chemistry. But they cannot only be found on the molecular level of matter. Atoms and elementary particles in physics are also characterized by particular symmetry groups. Even living organisms and populations on the macroscopic level have functional properties of symmetry. The whole physical, chemical, and biological evolution seems to be regulated by the emergence of new symmetries and the breaking down of old ones. One is reminded of Heisenberg's famous statement: «Die letzte Wurzel der Erscheinungen ist also nicht die Materie, sondern das mathematische Gesetz, die Symmetrie, die mathematische Form» (Wandlungen in den Grundlagen der Naturwissenschaften, 1959). Historically the belief in symmetry and simplicity of nature has a long philosophical tradition from the Pythagoreans, Plato and Greek astronomers to Kepler and modern scientists. Today, «symmetries in nature» is a common topic of mathematics, physics, chemistry, and biology. A lot of Nobel prizes were given in honor of inquiries concerning symmetries in nature. The fascination of symmetries is not only motivated by science, but by art and religion too. Therefore «symmetries in nature» is an interdisciplinary topic which may help to overcome C.P. Snow's «Two Cultures» of natural sciences and humanities.

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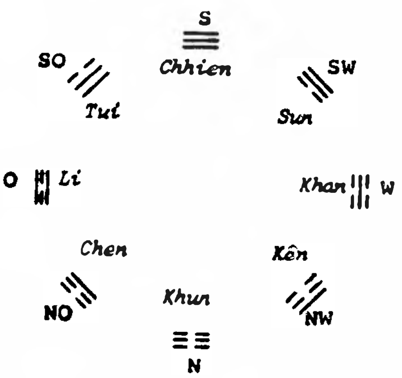
1. Remarks on Metaphors, Models, and Structures

In spite of an ever growing specialization, modern natural sciences intend to reduce their theories to some fundamental structures: *physics* tries to unify the different physical forces in one fundamental force; *chemistry* tries to explain the structure of chemical substances by the quantum mechanics of molecules; *biology* tries to reduce the processes of life to biochemical and biophysical laws.

Mathematically the unification of natural science can be described by structures of *symmetry*, the specialization of science, the variety and emergence of new phenomena by *symmetry breaking*. In the following I want to show 1) the successes and lacks of the *reductionistic program* by recent developments in physics, chemistry, and biology; 2) that the traditional philosophical discussion on *holism*, *reductionism*, and *unification of science* can be clarified by structures of symmetry and symmetry breaking.

Historically ideas of symmetry have been very influential in the development of human thoughts[1]. In technics people found out the practical advantage of balance. Pots and baskets of symmetric forms are more stable and economical than other ones. In agriculture people had to observe the periodic regularities of day and night, summer and winter, growth and death, etc. The regular movements of stars seemed to symbolize an eternal order of nature. No wonder that in mythologies, religions, and early cultures symmetric forms like circles were used as *metaphors* and symbols (Fig. 1). On the other hand people were fascinated by the beauty of regular patterns and ornaments.



I Ching

Chinese coin

Fig. 1. Symmetries as metaphors. In the Chinese natural philosophy of Taoism the fundamental phenomena of nature (heaven–earth, fire–water, thunder–wind, sea–mountain) were ordered on a circle in diametric positions. In the «Book of Wisdom» (I Ching) these phenomena were reduced to the dual forces «Yang» and «Yin» which were symbolized by an unbroken and broken stroke, respectively. In the everyday life of China these symbols were used on coins.

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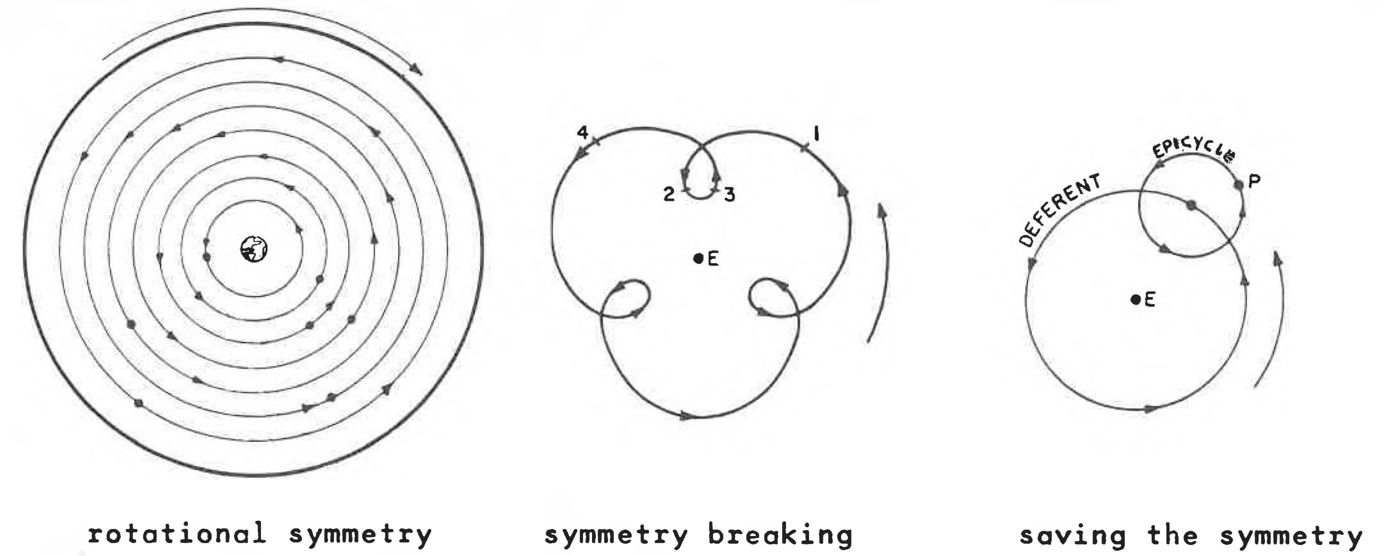

Fig. 2. Early symmetric models in Greek cosmology. The planets were at first assumed to move uniformly on the spheres of a geocentric model. Irregularities of planets which were observed later on were interpreted as a kind of symmetry breaking which had to be reduced to modified symmetric models: Philosophers and astronomers believed in the simplicity and symmetry of nature. Therefore the retrograde movements of planets were geometrically explained by epicycles the centres of which moving on deferents round the earth.

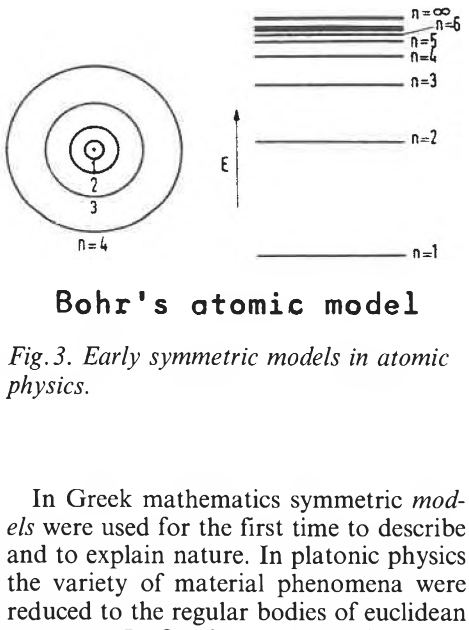

Fig. 3. Early symmetric models in atomic physics.

In Greek mathematics symmetric *models* were used for the first time to describe and to explain nature. In platonic physics the variety of material phenomena were reduced to the regular bodies of euclidean geometry. In Greek astronomy the movements on heaven were described by models of rotational symmetry. Irregularities of planets were interpreted as merely phenomenal symmetry breaking which was reduced to modified symmetric models (Fig. 2)[2].

Even in the natural sciences of modern times symmetric models were often used to illustrate and visualize natural regularities from *Kepler*'s heliocentric model of planets to *Rutherford*'s and *Bohr*'s model of atoms and electronic orbitals (Fig. 3).

Since the 19th century symmetries are not only defined as properties of geometric models, but as properties of natural laws and *theories*, too. In this sense symmetry means the invariance of a theory with respect to a transformation of its coordinates by a mathematical transformation group. This will be the main topic of my following considerations.

2. Mathematical Conception of Symmetry[3]

Mathematically symmetries are defined by so-called *automorphisms* (Fig. 4), i.e. self-mappings of figures, spaces etc. which let the structure invariant (example: rotation or reflection of polygons in the plane). The composition of automorphisms satisfies the axioms of a mathematical *group*. So the symmetry of a figure, body, or – in general – a *structure* is defined by its group of automorphisms. There are continuous groups of symmetries (for instance circles and spirals) and discrete groups (for instance regular polygons, ornaments, platonic bodies). We can distinguish *hierarchies* of more or less complex structures of symmetry (example: *F. Klein*'s «Erlanger Program» of geometrical transformation groups).

3. Symmetries of Space and Time[4]

The different conceptions of space-time, which were proposed in the natural philosophy from *Newton* and *Leibniz* until the 19th century, are mathematically more or less complex structures of symmetry. *Galilean invariance* means that the form of natural laws («equation of motion») in classical mechanics is preserved («invariant») with respect to transformations of the galilean group (i.e. galilean transformations of the coordinates in uniformly moving inertial systems). Intuitively it means that a natural law is true independently of a particular reference system of an observer (Fig. 5). In this sense *Einstein*'s special relativistic space-time is an extension to a richer structure of symmetry, the *lorentz-invariant Minkowskian geometry* which satiesfies the constancy of the speed of light.

Classical mechanics and special relativity are examples of *global symmetry*, i.e. the equations are invariant if *all* coordinates are transformed simultaneously. Analogously the form of a sphere is invariant with respect to a rotation if the coordinates of *all* points are changed with the same angle (Fig. 6a).

In *general relativity* the inertial systems are accelerated to each other, and observers seem to be influenced by gravitational forces. In our geometric language we may say the local deviations of the global symmetry (caused by accelerations) are compensated by fields of force which preserve the symmetry («form-invariance») of the gravitational law (*«local symmetry»*). Analogously there are distortions on the surface of a sphere by local changes of the coordinates. The form of the sphere is preserved («saved») by the assumption of forces. In short gravitational forces are introduced by the transition from global to local symmetry (Fig. 6b).

4. Symmetries in the Quantum World[5]

Quantum systems (atoms, electrons etc.) have *incompatible (non-classical) observables* (position, momentum etc.) which do not commute with each other and which have no definite eigenvalues in each state. Their *symmetries* are defined by the invariance of the corresponding Hamilton operators. Examples are the rotational symmetry of atoms or the permutation symmetry of electrons in an atom which are indistinguishable in the sense of the Pauli principle. In simple cases these

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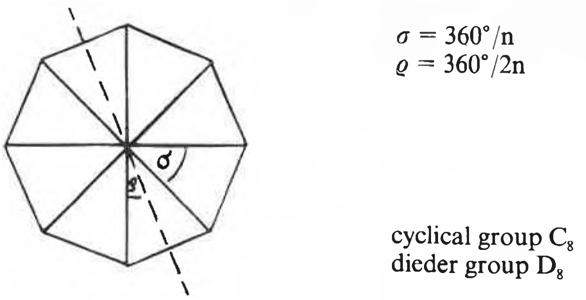
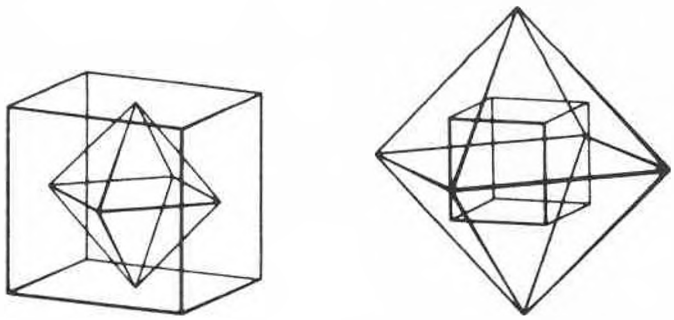
a) finite rotation groups:



b) platonic groups:



c) continuous group ('Lie group')

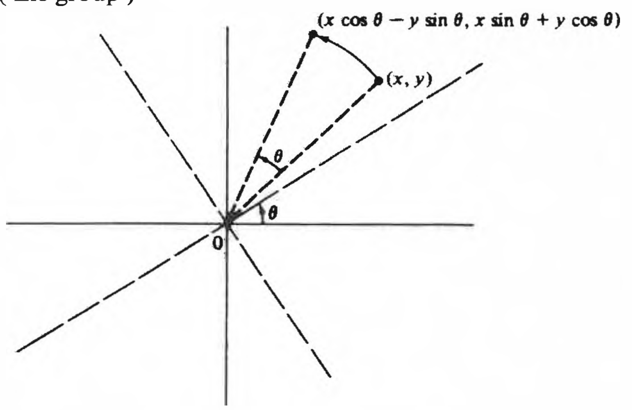
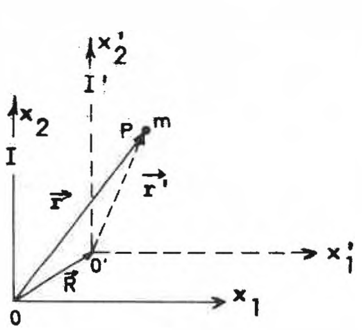


Fig. 4. Mathematical conception of symmetry. Symmetries are defined by automorphisms that means self-mappings of figures, spaces etc. which let the structure invariant. In geometry the mapping of similarity is an example of an automorphism which lets the form of a figure invariant. The relation of similarity $F \sim F'$ (figure F is similar to figure F') satisfies the conditions of an equivalence relation: 1) $F \sim F'$ (reflexivity); 2) if $F \sim F'$, then $F' \sim F$ (symmetry); 3) if $F \sim F'$ and $F' \sim F''$, then $F \sim F''$ (transitivity). In general the composition of automorphisms satisfies the axioms of a mathematical group A: 1) the identity I which maps a figure on itself is an element of A ($I \in A$); 2) for every mapping $T \in A$ there is an inverse $T^{-1} \in A$ with $T \cdot T^{-1} = T^{-1} \cdot T = I$; 3) if S and T are automorphisms, then the composition $S \cdot T$ is an automorphism. Examples of discrete groups are the finite rotation groups of polygons (Fig. 4a) and the platonic groups (Fig. 4b) which preserve the symmetric structure of the platonic bodies. An example of a continuous group is the rotation $R(\theta)$ with the continuous parameter θ (Fig. 4c) which satisfies the axioms of a group with $R(\theta_1 + \theta_2) = R(\theta_1) \cdot R(\theta_2)$, $R(0) = I$, $R(\theta)^{-1} = R(2\pi - \theta)$, $R(\theta) \cdot R(\theta)^{-1} = R(\theta)^{-1} \cdot R(\theta) = I$.



$$m \frac{d^2\vec{r}}{dt^2} = m \frac{d^2\vec{r}\,'}{dt^2} = \vec{K}$$

Fig. 5. Galilean invariance of classical mechanics. An inertial system I' moves uniformly relative to an inertial system I. The equation of motion is galilei-invariant, because it preserves its form after a galilei-transformation (spatial translation, rotation, temporal translation, translative velocity).

structural symmetries can be visualized at least approximately by geometric models, but not in general.

The main difference between classical and quantum systems is the following: Quantum systems which once have interacted remain in statistical correlations, even when they are separated with far distances without any dynamical interaction. This is a mathematical consequence of the so-called superposition principle of quantum mechanics, which is today well confirmed by the EPR (= *E*instein-*P*odolski-*R*osen)-experiments of *Aspect* in 1981/82 (*«EPR-correlations»*). In short quantum mechanics with an unrestricted superposition principle describes a whole which is not made out of isolated parts (Fig. 7).

This *unbroken wholeness* of the quantum world is mathematically defined by the *logical symmetry* of the quantum world. In more technical words it is given by the automorphism group Aut($\bar{H}$) of the projective Hilbert space $\bar{H}$ (associated with the Hilbert space H) which corresponds to the states of a quantum system. A famous theorem of *Wigner* (1931) asserts that the automorphism group Aut($\bar{H}$) can be represented by the group of unitary operators on the state space H[6].

The *space-time structure* of a quantum system can be specified by a *subgroup* of the logical symmetries of quantum mechanics. In more technical words the galilean-invariance of quantum mechanics is given by a projective unitary representation of the Galilei group on the Hilbert space of state vectors[7].

The idea of a total wholeness may be strange to grasp for scientists educated in the *classical atomistic tradition* who are accustomed to accept the separability of objects as something self-evident. From the viewpoint of quantum mechanics this is a highly artificial and only approximative doctrine, as we shall see later on.

5. Dynamical Symmetry and Symmetry Breaking[8]

Nowadays physics distinguishes four fundamental forces: the electromagnetical, strong, weak, and gravitational force (Fig. 8). They can be introduced by a transition from *global* to *local symmetry* (as in the case of the gravitational force). Forces are interpreted as so-called *gauge fields* which compensate local deviations of a global symmetry.

In *electrodynamics* a magnetic field compensates a local change of an electric field, i.e. the movement of a charged body, and preserves («saves») the invariance of electromagnetic field equations. In *quantum electrodynamics* an electromagnetic field compensates the local change of a material field (phase deviation of an electronic field) and preserves («saves») the invariance of the corresponding field equations.

In a *two-slits experiment* with an electronic wave the phase deviation at both slits is a global change which delivers a global symmetry of the electronic field (Fig. 9a). A phase deviation at only one slit is a local change which can be compensated for instance by a magnetic field (Fig. 9b). Mathematically the phase deviations are described by transformations $\psi \rightarrow e^{i\alpha} \psi$ with a unitary 1 × 1-matrix as phase factor. So the electromagnetic force is defined by a *local U(1)-symmetry*.

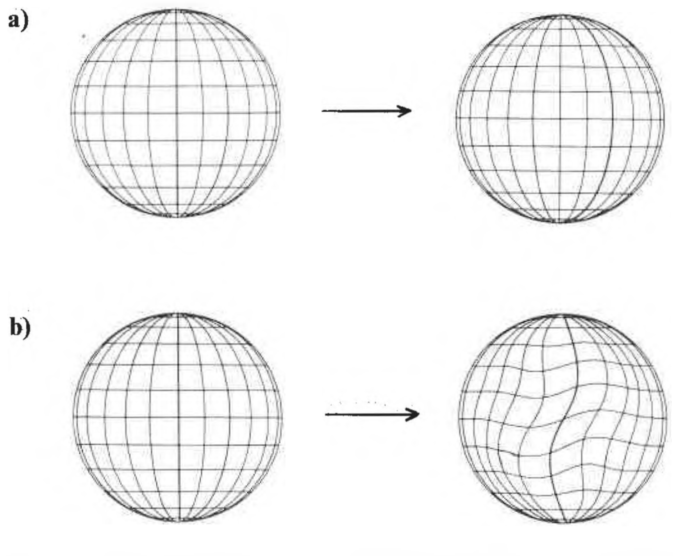
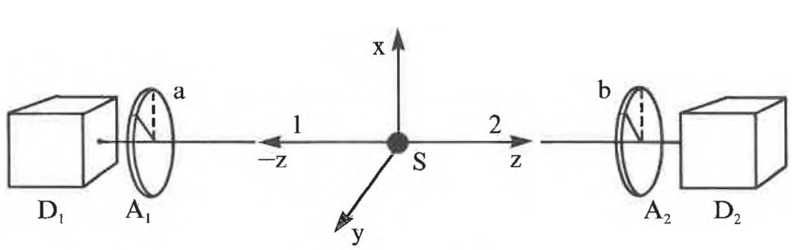


Fig. 6. a) Global symmetry: Equations of motion («natural laws») are form-invariant if all coordinates are transformed simultaneously. Analogously the form of a sphere is invariant after a rotation if the coordinates of all points are changed with the same angle. – b) Local symmetry: The local deviations of the global symmetry (caused by accelerations) are compensated by fields of force which preserve the form-invariance (symmetry) of gravitational equations. Analogously there are distortions on the surface of a sphere by local changes of the coordinates. The form of the sphere is preserved by the assumption of forces. We may say that in general relativity the gravitational forces are introduced by the transition from global to local symmetry.



EPR-experiment

Fig. 7. Holism of the quantum world: Quantum systems (photons, protons etc.) which once have interacted remain in EPR-correlations even when they are separated with far distances without any dynamical interaction. In the famous experiments of A. Aspect (1981) pairs of photons are scattered from a source S in the opposite z- and −z-direction. Their EPR-correlations can be confirmed by the analysis of polarization states in A_i and detectors D_i.

	gravitation	electromagnetism	strong force	weak force
reach	∞	∞	10^{-13}–10^{-14}	10^{-14}
example	forces between astronomical objects	forces between charges (in an atom)	connection of nuclei	β-decay of atomic nuclei
units	$G_{Newton} = 5.9 \times 10^{-39}$	$e^2 = 1/137$	$g^2 \approx 1$	$G_{Fermi} = 1.02 \times 10^{-5}$
particles	all	charged particles	hadrons	hadrons and leptons

Fig. 8. Fundamental physical forces.

The complex variety of particles like hadrons (protons, neutrons etc.) which interact with *strong forces* can be reduced to the so-called quarks with three degrees of freedom, i.e. the «colors» red (R), green (G), and blue (B). A baryon is built up by three quarks which are distinguishable by three different colors. These three colors are complementary in the sense that a hadron is neutral («without color») to its environment. The color state of the hadron preserves invariance with respect to a *global transformation* of the colors (Fig. 10a). But a *local transformation* of a color state (i.e. a color change of only one or two quarks) needs a gauge field, in order to compensate the local change and to save the invariance (symmetry) of the whole hadron (Fig. 10b). Mathematically we have a local so-called *SU(3)-symmetry*.

Elementary particle physics intends to unify the four physical forces in one fundamental force. Electromagnetic and weak forces could already be unified by very high energies in an accelerator ring of CERN. It means that at a state of very high energy the particles of weak interaction (electrons, neutrinos etc.) and electromagnetic interaction cannot be distinguished. They can be described by the same *symmetry group* U(1) × SU(2). At a particular critical value of lower energy the symmetry breaks down in two partial symmetries U(1) and SU(2) which correspond to the electromagnetic and weak force.

The process of *spontaneous symmetry breaking* is well-known in physics. For instance your breakfast egg is not stable in its symmetric position (Fig. 11a). Caused by a tiny fluctuation it falls spontaneously down to an asymmetric, but energetically stable position. The phase transition of a ferromagnet from a non-magnetic to a magnetic state is caused by cooling down the temperature to a critical point (Fig. 11b). In this case two magnetic orientations are possible («north» and «south»). Spontaneously the elementary dipols decide for one possibility and break the spin-rotation symmetry.

After the successful unification of electromagnetic and weak interaction physicists try to realize the *«big» unification* of electromagnetic, weak, and strong forces, and in a last step the *«superunification»* of all four forces (Fig. 12). Technically the unification steps should be realized with growing values of very high energy. Mathematically they are described by extensions to richer structures of symmetry («gauge groups»). On the other hand the *variety* of elementary particles can be actualized by symmetry breaking.

In the *cosmic evolution* (Fig. 13) there was at first (for a short moment after the «Big Bang») a fully symmetric situation

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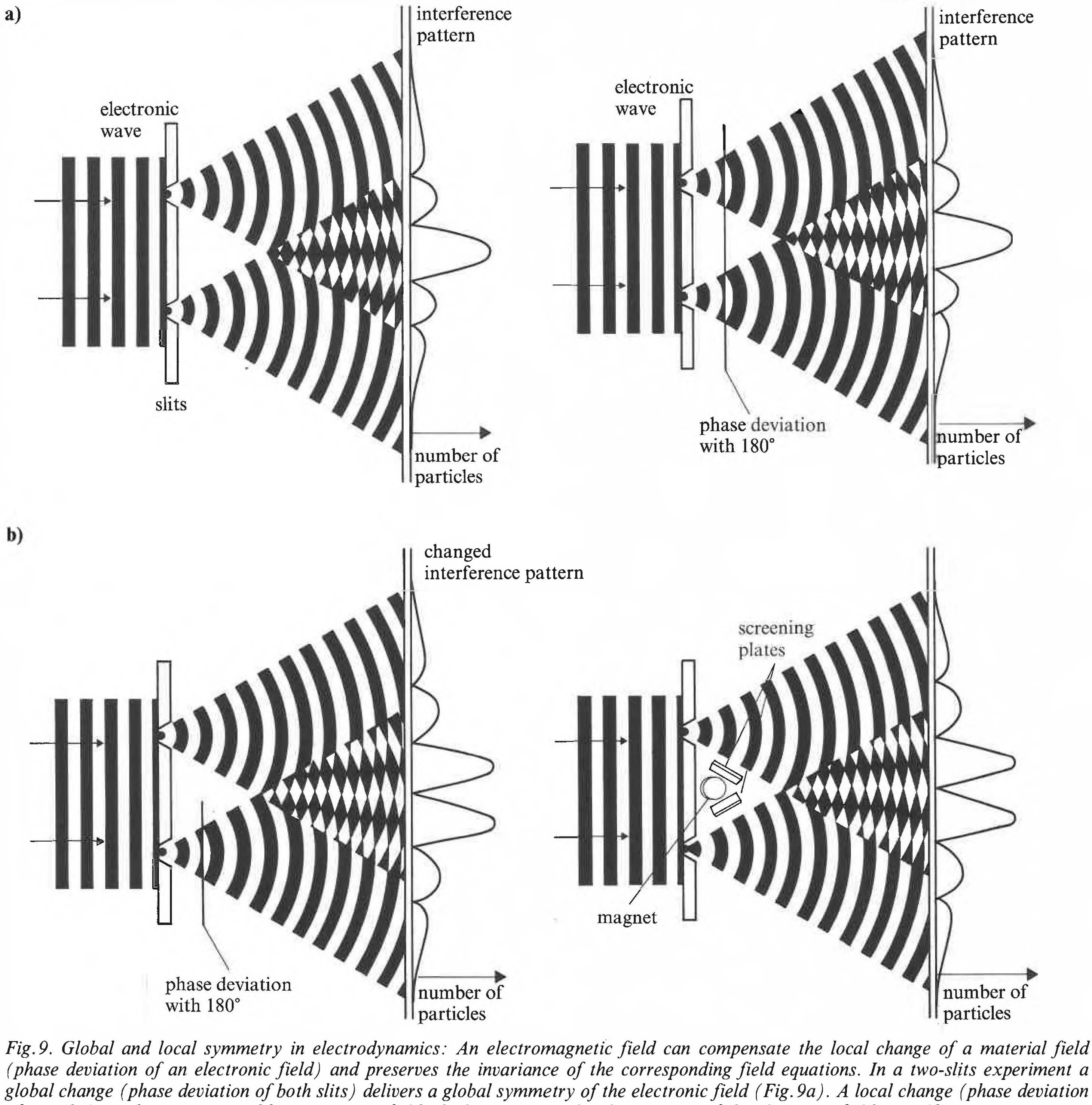

Fig. 9. Global and local symmetry in electrodynamics: An electromagnetic field can compensate the local change of a material field (phase deviation of an electronic field) and preserves the invariance of the corresponding field equations. In a two-slits experiment a global change (phase deviation of both slits) delivers a global symmetry of the electronic field (Fig. 9a). A local change (phase deviation of one slit) can be compensated by a magnetic field which preserves a local symmetry of the electronic field (Fig. 9b).

of very high energy in which no particles can be distinguished, but they all can be transformed into one another. During the retardation of the cosmic evolution and cooling of its temperature, critical values were realized step by step at which symmetries break down and new particles and forces emerge: «C'est la dissymétrie, qui crée le phénomène», said *Pierre Curie* [9].

It is a special property of the *weak force* which violates the *parity* of its particles: Only left-handed electrons and neutrinos take part in weak interactions and can be described by the isospin-symmetry SU(2).

6. Molecular Symmetries and Symmetry Breaking

The variety of elementary particles and forces can be distinguished by different dynamical symmetries, but they all are correlated in the unbroken wholeness of EPR-correlations which is described by the logical symmetries of quantum mechanics and quantum field theories. The reductionistic program of chemistry demands that chemical substances and molecules are built up by these elementary building blocks and consequently can be explained by the principles of quantum mechanics.

But now a severe problem arises: Chemists describe their molecules with geometric models and structural formula. They even distinguish different electronic orbitals which are absolutely impossible in quantum mechanics because of the Pauli principle. So chemists use *classical observables* like shape, orbitals etc. with definite eigenvalues, while in quantum mechanics (with unrestricted superposition principle) we only have quantum systems with incompatible (non-classical) observables and indefinite values. This is the reason why chemists can regard molecules, atoms, and electrons as isolated

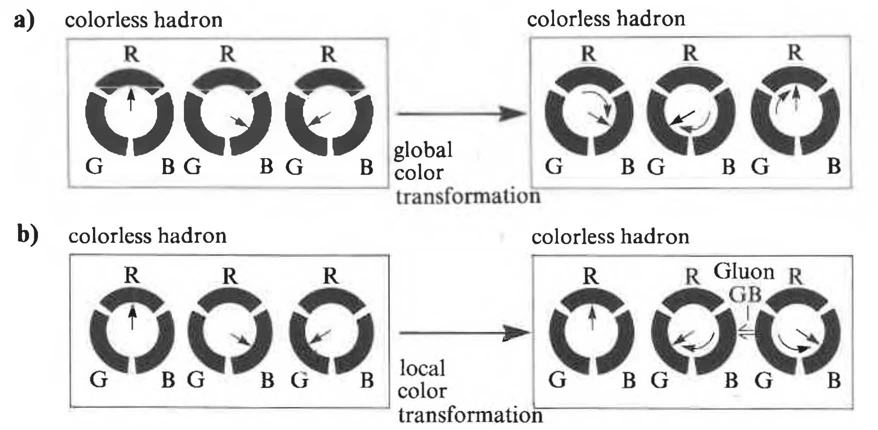

Fig. 10. Global and local symmetry in the chromodynamics of strong forces: A global color transformation preserves the color state of a hadron (baryon) invariant («colorless»), because the color state of all quarks is changed with the same «angle» (Fig. 10a). Color-gauge fields compensate local changes and preserve the symmetry (Fig. 10b).

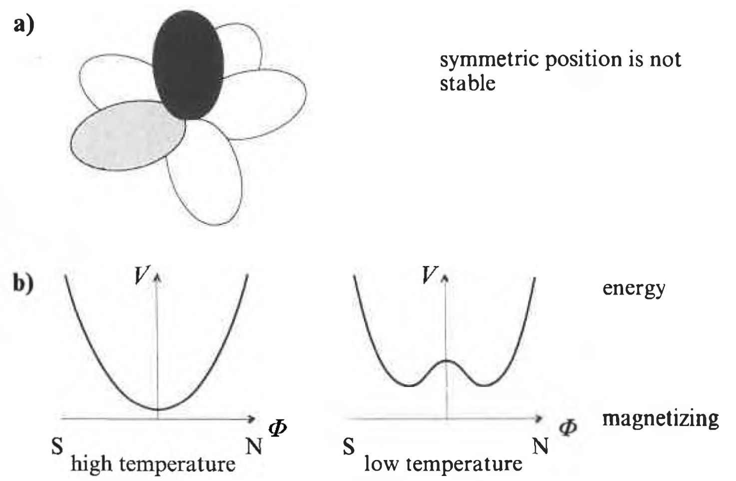

Fig. 11. Spontaneous symmetry breaking: Changing some critical conditions of a system causes a transition from a symmetric (but energetically instable) state to an asymmetric (but stable) state. Examples: The symmetric position of an egg is not stable (Fig. 11a). The phase transition of a ferromagnet from a non-magnetic to a magnetic state is caused by cooling down the temperature (Fig. 11b): the elementary dipols decide for one of two possible orientations and break the spin-rotation symmetry.

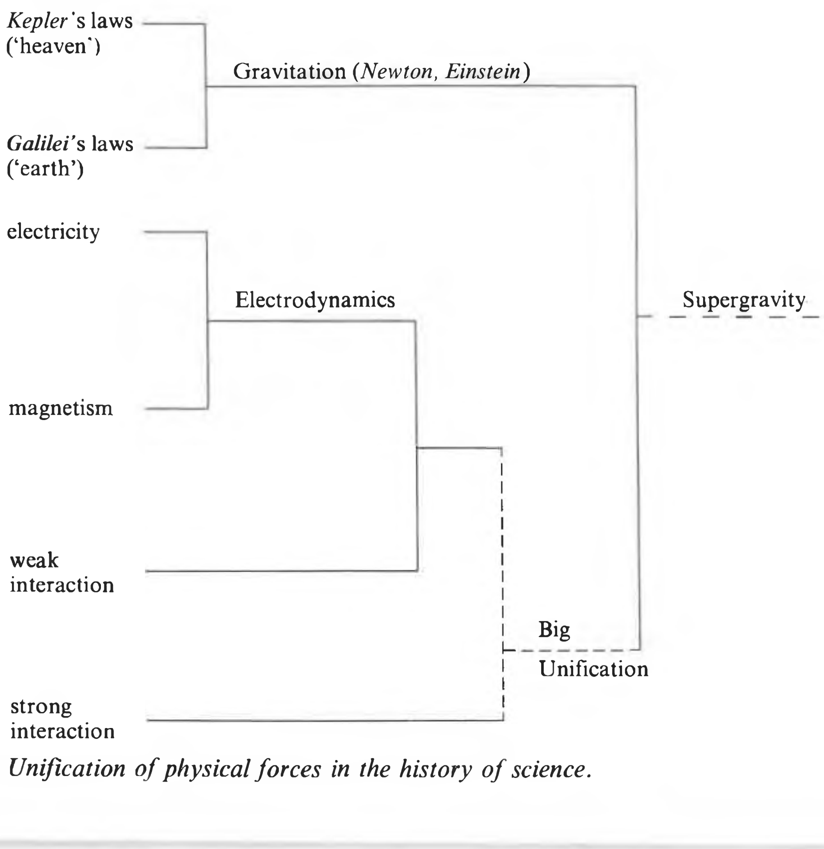

Fig. 12. Unification of physical forces in the history of science.

building blocks with well-defined properties, but quantum physicists not. What does it mean that physical and chemical electrons are different? Is there an insurmountable boundary between physics and chemistry?

In order to introduce classical observables and to isolate molecular models in the quantum world, we have to add some abstractions to the principles of quantum mechanics which suppress the factually existing EPR-correlations. This means the resulting *quantum chemistry* of molecules breaks the whole symmetry of quantum mechanics and is mathematically given by a particular *subgroup* of the full group of all *logical symmetries* of quantum mechanics.

Philosophically we must be aware that abstractions are epistemic actions which deliver new patterns and structures. *Pattern recognition* is now made precise by mathematical symmetry breaking. Without making any abstractions from EPR-correlations the quantum mechanical wholeness is observationally empty. Scientists like chemists have some special interest to investigate a particular part of nature and to manage chemical experiments. So they reduce the complexity of the whole theory and create new facts with abstraction and symmetry breaking.

Since *L. Fleck* (1935) and *T.S. Kuhn* it is well known in philosophy of science that every *fact* is *conditional* and *context-dependent*. What a person or society means by a «fact» cannot be understood in isolation from the whole conceptual structure. For every problem, for every observation, for every experiment we have to choose the appropriate context. The concept of symmetry and symmetry breaking makes the psychological remarks of *Fleck* and *Kuhn* mathematically precise and applicable in natural science[10].

It may be helpful to summarize the main steps of abstractions which are necessary to get the molecular viewpoint from the principles of quantum mechanics (Fig. 14)[11]. In a first step we have to specify the *space-time structure* in which we would like to discuss molecular systems. In chemistry the galilean space-time is usually appropriate, because we may neglect high velocities like light. Mathematically this can be done by contracting the Lorentz group to the Galilei group ($c \to \infty$). So we abstract from the factually existing EPR-correlations between electrons and positrons. We get the mathematical consequence that mass is now a classical observable[12].

Next we have to isolate a *molecular system* from its *environment* by abstracting from the factually existing EPR-correlations between the selected object system and its radiation field. Such a molecular system is characterized by its molecular formula and mass. But it is not a molecule in the sense of traditional chemistry, because it has no shape and no intrinsic

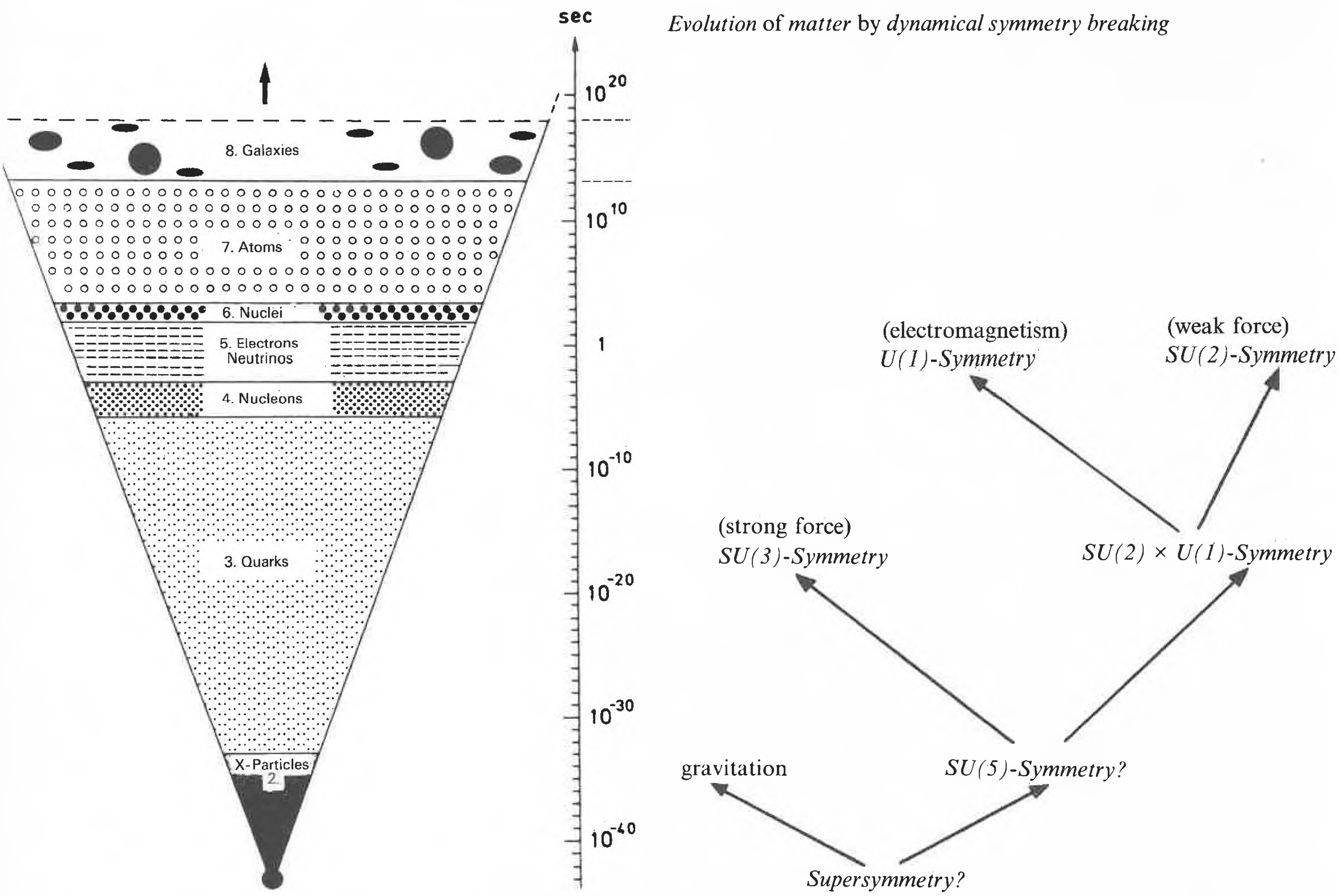

Fig. 13. Symmetries and symmetry breaking in the cosmic evolution: The evolution of matter and forces since the «Big Bang» can be interpreted by the process of symmetry breaking in the quantum field theories. During the retardation of the cosmic evolution and cooling of its temperature, critical values were realized step by step at which symmetries break down and new particles and forces emerge.

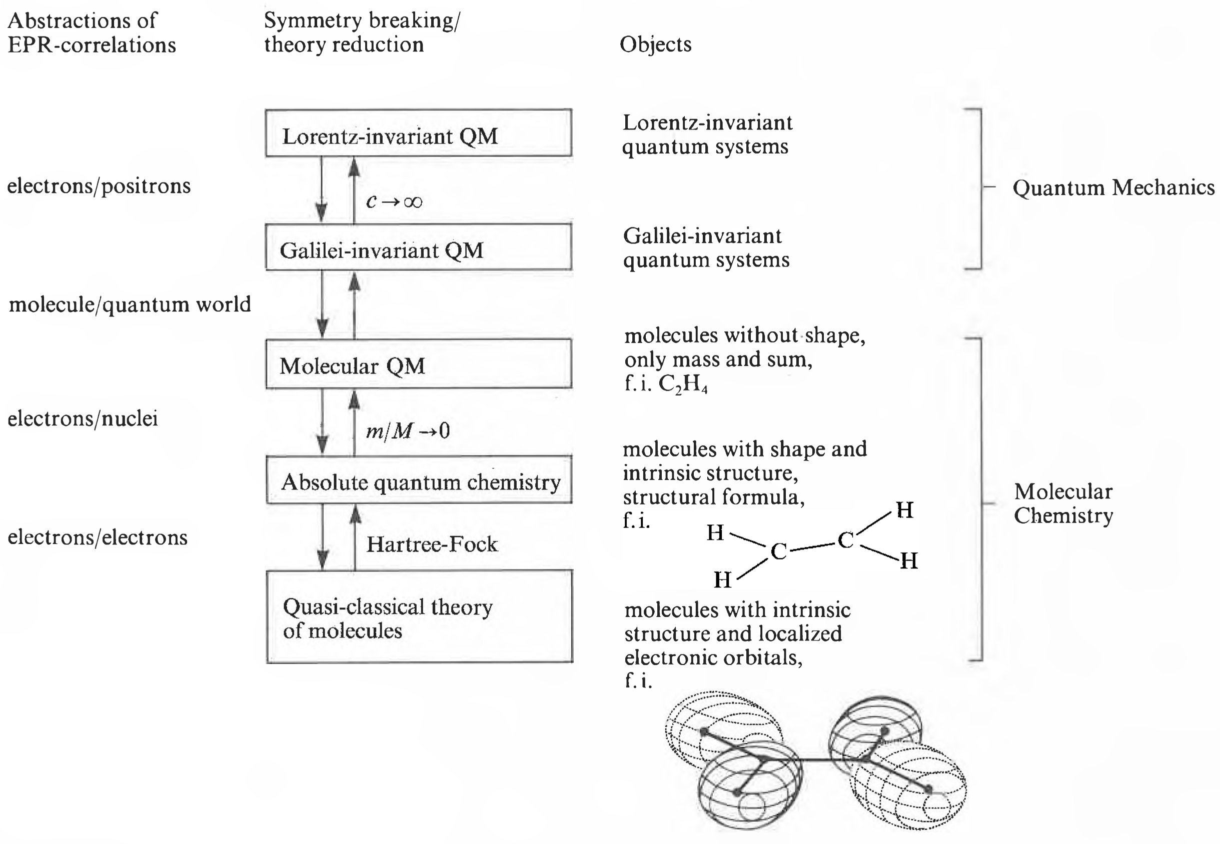

Fig. 14. Emergence of chemical structures by symmetry breaking or weak theory reduction.

molecular structure. In short it has no structural formula.

In order to get a visualized geometric model of chemistry, one has to abstract from the factually existing EPR-correlations between the nuclei and the electrons. In chemistry this breaking of a logical symmetry is realized by the so-called *Born-Oppenheimer procedure*. In mathematically more technical words we have to regard an asymptotic expansion $m/M \to 0$ with electronic mass m and a mean nuclear mass M. We get some new classical observables like the molecular *shape* and geometric *structure* which emerge with this abstraction and approximate procedure.

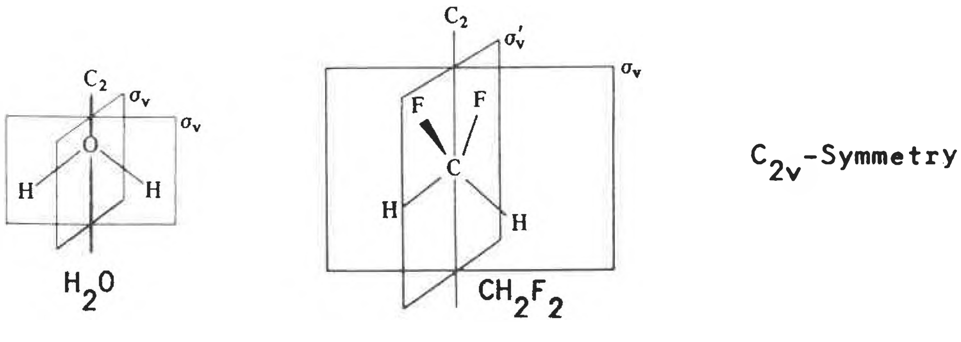

Fig. 15. Symmetry of free molecules: Different molecules can have the same point group of symmetry.

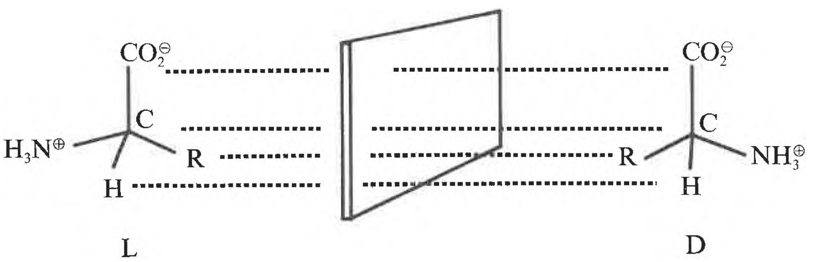

Fig. 16. Dissymmetry («homochirality») of macromolecules: Left-handed α-amino acid as building block of proteins.

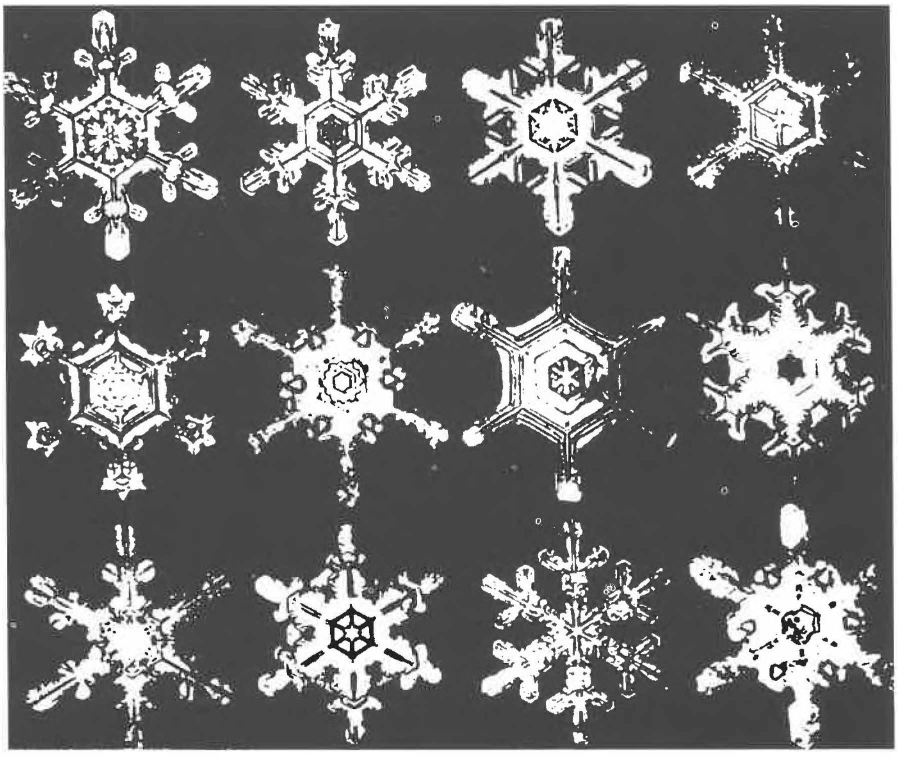

Fig. 17. Symmetry breaking of macroscopical systems in thermal equilibrium: In ferromagnets we observe a transition from non-magnetic to magnetic states by cooling down the temperature which breaks the symmetry of the dipols (Fig. 11b). In snow crystals we observe a transition from non-crystalized states to crystalized states by cooling down the temperature which breaks the symmetry of the homogeneous distribution of molecules.

If we wish to separate even the *electrons* on localized orbitals as quasi-classical objects, we have to neglect the EPR-correlations between the electrons, too. In chemistry this breaking of a logical symmetry is realized for instance by the *Hartree-Fock* approximation.

On the one hand the way from the holistic quantum world to the molecular models with less logical symmetry and structure is a *weak* (approximate) *theory reduction*. On the other hand the chemical models have a richer structure, because *new properties* («observables») and *objects* emerge with symmetry breaking. Single molecules or crystals may have rich partial symmetries which can be described by mathematical point groups and compared with different degrees of complexity (Fig. 15).

In *biochemistry* macromolecules (for instance L-amino acids or D-sugars) which are building blocks of living systems possess a characteristic *homochirality* (*«dissymmetry»*) which is assumed to be caused by parity violation of weak physical forces (Fig. 16)[13]. In the 19th century *Pasteur* already presumed that living systems are characterized by typical dissymmetries of their molecular building blocks. Looking at the rigid beauty of snow crystals *Thomas Mann* led his hero, Hans Castorp, in the novel «Zauberberg» say that life is afraid of cold symmetry: «Dem Leben schauderte vor der genauen Richtigkeit».

7. Biological Symmetries and Symmetry Breaking

The emergence of pattern structure can be described by symmetry breaking not only in chemistry, but even in biology. Since the pioneering work of the famous English logician and mathematician *A. Turing* on the chemical basis of *morphogenesis* in biology (1952), there is an increasing interest in this topic[14]. The spontaneous emergence of macroscopic structures is well known in *thermodynamics*. We already mentioned the magnetism of a ferromagnet. Another example is the structure of snow crystals which suddenly arise in a homogeneous and symmetric situation, when temperature is cooled down to particular values. These are examples of systems in *thermal equilibrium* with their environment (Fig. 17).

In systems *far from thermal equilibrium* patterns can arise suddenly if the input of energy increases to particular values and establishes a permanent metabolism with their environment (Fig. 18)[15]. Chemical examples are the *dissipative structures* which suddenly arise in homogeneous mixtures (Zhabotinsky reaction). A famous physical example is the *laser light*

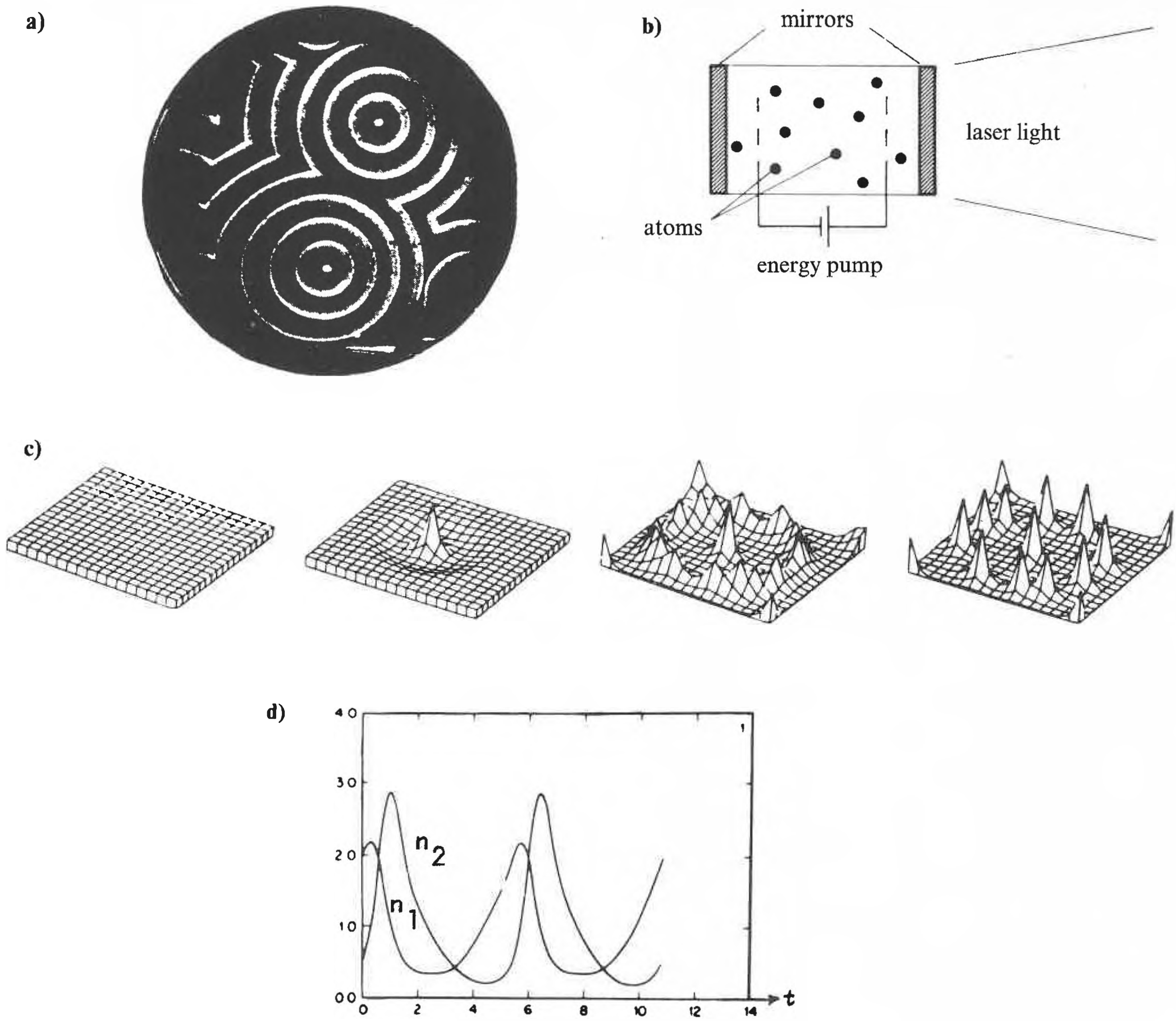

Fig. 18. Symmetry breaking of macroscopical systems far from thermal equilibrium: In open complex dynamical systems far from thermal equilibrium new patterns and structures can arise suddenly if the input of energy increases to critical values and establishes a permanent metabolism with the environment. Their phase transition is mathematically described by non-linear equations of evolution. Examples are the dissipative structures in chemistry (Fig. 18a: Zhabotinsky reaction), the laser light in physics (Fig. 18b), and the morphogenesis in biology (Fig. 18c). The cell differentiation of growing organisms is understood as a kind of symmetry breaking which can be simulated by computers. Even the ecological balance of populations can be understood by non-linear phase transitions of open dynamical systems (Fig. 18d: Lotka-Volterra equations of predator fishes n_2 and preyfishes n_1).

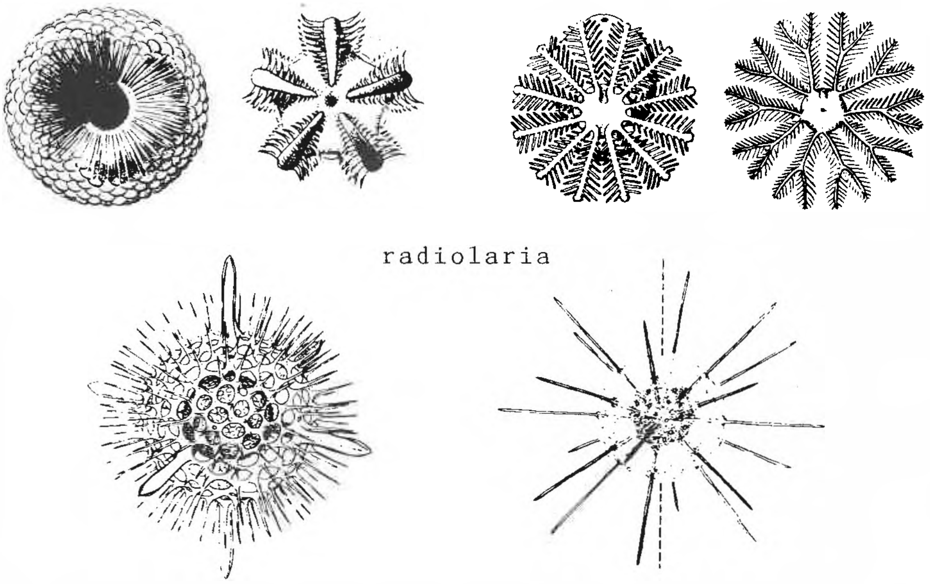

Fig. 19. Rotational symmetries of organisms (D'Arcy Thompson, E. Haeckel).

which suddenly breaks the distribution of emitted photons in an active material if the pump energy arises a particular value.

Especially *living organisms*, which are in *metabolism* with their environment, are systems far from thermal equilibrium. The morphogenesis of these systems can be described by the same methods of symmetry breaking. A well understood example is the growth of a fungiform organism (*Dictyostelium disciodeum*) which suddenly breaks the symmetry in an aggregation of equal (equipotent) cells with homogeneous distribution if the nutrition of the cells becomes critical and is no longer secured. The symmetry breaking causes a typical cell differentiation.

Even the growth of *macroscopic populations* (i.e. animals) can be described by symmetry breaking. A population is understood as a dynamical system the

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growth of which is in an *ecological balance* with its environment until this symmetry is broken by some irreversible disturbance. The symmetric pattern can be visualized by the rhythmical curves of the corresponding population equations (for instance the Lotka-Volterra equation of predator fishes and their preyfishes).

In the theory of *evolution* the growth of organic forms and populations is interpreted as *functional* development, i.e. as an optimal adaption to the conditions of environment (Fig. 19 and 20). Mathematically all these examples can be understood as dynamical systems the growth of which is determined by *non-linear equations*. At a first glance the non-linearity of these macroscopic systems seems to be an insurmountable difference to the linearity of microscopic quantum systems (superposition principle). Concerning our question of reductionism one can get non-linear evolution equations out of quantum mechanics by approximate decorrelation assumptions (factorization of expectation values, neglect of higher-order correlations etc.). In this sense the spontaneous symmetry breaking of non-linear systems can be understood at least in principle for models of lasers in quantum optics. But in detail the variety and complexity of macroscopical systems is very difficult to be explained in the framework of quantum mechanics from a microscopic point of view[16].

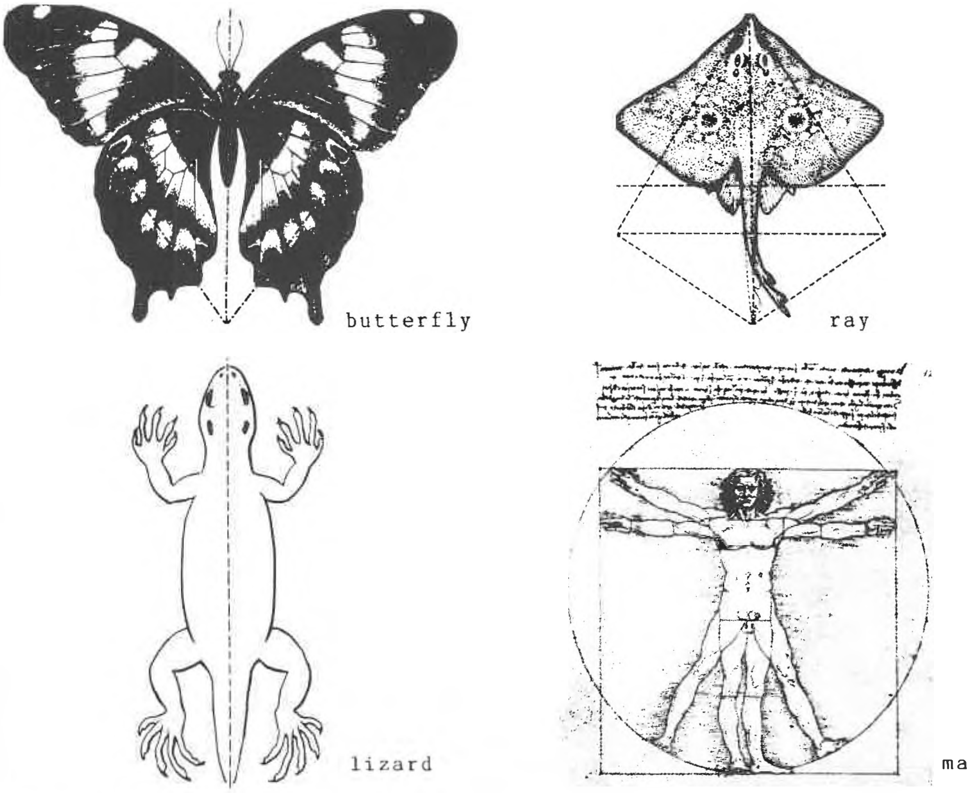

Fig. 20. Bilateralia of animals.

THEORY REDUCTION ▲ / SYMMETRY BREAKING ▼

THEORY	OBJECTS	SYMMETRIES
quantum field theories	elementary particles, forces	logical symmetries of quantum systems: f.i. Aut($\bar{H}$)
		kinematical space-time symmetries: f.i. Galilei-, Lorentz-group
		dynamical symmetries: f.i. SU(2) × U(1)-, SU(3)-forces
quantum chemistry	atoms, molecules	structural-, orbital-, crystal-symmetries
chemistry	etc.	
biochemistry	macromolecules	homochirality
thermodynamics	open systems with metabolism	dissipative structures
biology	organisms	functional symmetries
ecology	populations	ecological balance

Fig. 21. Summary of symmetries and symmetry breaking in the hierarchy of nature and natural sciences.

8. Holism, Reductionism, and Symmetry Breaking in the Philosophy of Science[17]

It may be worthwhile now to summarize our run through the hierarchy of natural systems (Fig. 21). We start with the framework of *quantum field theories* and the principles of *quantum mechanics*. It describes a *holistic quantum world* with *logical symmetries* which are represented by EPR-correlations. *Kinematical space-time symmetries* are only special cases (subgroups) of the general logical symmetries. Elementary particles and forces can be distinguished by the symmetry breaking of *dynamical symmetries* (gauge groups), although they remain correlated in the EPR-wholeness of the quantum world. Dynamical symmetry breaking can be understood as cosmological evolution of particles and forces which has caused the variety of the material world in its present state.

From this physical basis we get the hierarchically higher level of molecules, chemical compounds, crystals etc. by several steps of abstraction which can be interpreted as weak (approximate) theory reduction of *chemistry* to quantum mechanics. On the other hand it is a symmetry breaking of some logical symmetries in the quantum mechanical wholeness which is connected with the emergence of new patterns and partial symmetries (intrinsic molecular structure, electronic orbitals, crystal symmetries etc.) which does not exist at the lower level of the hierarchy. In this sense these steps of symmetry breaking deliver an extension to richer structures.

In *biochemistry* we get a hierarchically higher level of macromolecules with new partial symmetries and symmetry breaking which are possible building blocks of living systems.

Even the emergence of macroscopic systems can be described by this process of structural extension which is realized by symmetry breaking. In *thermodynamics* we mentioned open systems far from thermal equilibrium which are in metabolism with their environment and which achieve new dissipative structures by spontaneous symmetry breaking. The morphogenesis and functional forms of *organisms* can also be understood as emergence of new patterns and structures which are realized by symmetry breaking.

Even the hierarchically higher level of *populations of organisms* in ecology can be introduced by symmetry breaking. Mathematically these macroscopic dynamical systems are described by non-linear equations of evolution which at least in principle can be reduced to the linear principles of the quantum mechanical framework.

A description of nature in terms of *hierarchical symmetry structures* and symmetry breaking seems to be appropriate to grasp the diversity and complexity even of biological systems. So a natural but still reductionistic extension of elementary particle physics and molecular chemistry would be in direction towards a hierarchical chemistry and biology. But we have to be careful not to confuse reality and our *description of reality*. It is the theory which is hierarchical and not nature. An ontological assumption of a metaphysical hierarchy in nature is not involved. This view has a lot of advantages: The transition from micro- to macroworld and from quantum to classical systems can be described in one general theoretical framework with several levels.

Lower and higher levels in hierarchical systems are characterized by different *time scales*, a higher level having a much larger reaction time than all lower levels. A hierarchically higher level is characterized by the *emergence of new qualities* with symmetry breaking. But at the same time a higher-level theory has a more restricted domain of validity and is less accurate than a more fundamental level.

In this framework it is true that a description of the typical phenomena of some hierarchical level is possible with a *language* belonging to a lower level. But such a description may be very complex and almost incomprehensible. So we should say that the upper-level entities have an existence in their own right and can be investigated in their own right. Every upper-level description requires its own language, because a complete translation of the language of a higher-level theory into the terminology of the more fundamental theory is neither possible nor desirable.

Alternative *non-reductionistic viewpoints* are probably possible and desirable for the lacks in the reductionistic program. For example *chemistry* is not only the science of molecules but also the science of substances which deals with the properties and the behavior of all kinds of matter. The observables of molecular chemistry are not appropriate for all practical descriptions of the behavior of complex macroscopical chemical systems (temperature, chemical potential etc.), although it is perhaps theoretically possible.

In the same sense *biology* is not only biochemistry and biophysics. The physiological and functional aspects of living systems can be described in a qualitative sense, although it is perhaps possible to reduce them at least in principle to molecular processes.

The great advantage of a *hierarchical view* which is factually maintained by natural scientists is the common framework in which physical, chemical, and biological aspects of nature can be understood. Symmetry and symmetry breaking is the *fundamental category* of this framework to which the usual categories of natural science like space, time, causality, interaction, matter, force, shape etc. can be reduced in a logical and mathematical precise manner. But this *categorical framework* is not understood as absolutely and necessarily apriori with a unique claim to legitimacy in the sense of *Kant*, but as a successful and consistent framework of research.

It offers new phenomena, new problems, and new problem solutions. It shows new connections between disciplines which were regarded as separated and isolated fields of research. So it supports *interdisciplinary work* and gives new insight in a *common structure* and *theory of natural science* – no more and no less. Last not least the principle of symmetry shows that even in modern research there are some leading philosophical ideas which date back to early times of mankind and which have been fertile during the long history of human thinking.

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