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Methodology For Robust Control Of Pressure For ε-Caprolactone Polymerization in a Twin Screw Extruder

Samir Choulak[a], Françoise Couenne*[a], Gerard Thomas[a], Philipe Cassagnau[b], and Alain Michel[b]

Abstract: In this paper we present a control system based on robust control, which was developed for the regulation of the pressure gradient at the output of a die. In addition, to model the dynamics of the process, we have chosen a second order model with dead time and the identification of dynamical parameters is performed by using the Marquard-Levenberg optimization method. However, to cover the entire range of operation, we were forced to consider two models. The results of the simulation obtained with the robust control are satisfactory.

Keywords: Identification · Reactive extrusion · Robust control

1. Introduction

For the past few years, continuous polymerization and in particular reactive extrusion has become a topic of study for many researchers due to the interest that it brings on the technical and economic level.

However, the control of such polymerization process requires the knowledge of complex coupled phenomena: the flow of non-Newtonian fluids inside the extrusion machine, the reaction in question, and heat exchange. It is thus currently difficult to propose 'realistic' and simple models that represent the phenomenon as a whole.

The considered approach consists of the development of a simple model based on the knowledge of the theoretical stationnary model of the pressure ΔP_{theo}, and to present a process control of the quality of the polymer at the output of the extrusion machine by using the measurement of the gradient of pressure at the output of the die. The regulation of this pressure is carried out by acting on the monomer/initiator ratio ($[M]/[I_0]$) of the reactive mixture supplying the extrusion machine. The other parameters of adjustment were kept fixed, namely the temperature of the sleeve, the disk speed of the screws and the global feed rate.

2. Identification of the Process

The goal of the identification is to estimate the dynamics of the open loop transfer function between the pressure gradient in the output of the die and the theoretical stationary pressure which is given in [1] by Eqn. 1, 2.

From the experiments carried out, we have noted that a second order dead-time transfer function gives a satisfying representation of the process. This model characterizes the dynamic behavior of the process.

Since the process is highly non-linear, it was not possible to obtain one transfer function between the measure ΔP and the input ΔP_{theo} (with dead time). Thus, a multi-model strategy was chosen. For a given speed screw (100tr/min), a global feed rate (3 kg/h), a constant profile of temperature along the extruder (170 °C), two models were necessary to represent the process on the class of ratio $[M]/[I_0]$ 300 to 900.

$$\Delta P_{mes1}(s)/\Delta P_{theo}(s)=e^{-100s}/(a_1 s+1)^2 \quad (3)$$

for [M]/[Io] varying between 300 and 600

$$\Delta P_{mes2}(s)/\Delta P_{theo}(s)=e^{-100s}/(a_2 s+1)^2 \quad (4)$$

for [M]/[Io] varying between 600 and 900.

Moreover, some parameters of the non-linear function $\Delta P_{theo}([M]/[I_0], T)$ have to be adjusted for each model: the power coefficient of the molecular

$$\Delta P_{theo}([M]/[I_0],T)=\left(8L/\pi R_1^3\right)\cdot Q\cdot\left[\left(\overline{\eta}_0 M_w^{a}\right)\Big/\left[1+\left[\left(4Q/\pi R_1^3\right)\cdot k_1\cdot\left(\overline{\eta}_0 M_W^{\Omega}\right)^{0.9642}\right]^{a}\right]^{(1-n)/a}\right] \quad (1)$$

$$\text{with}: \overline{\eta}_0=k\cdot e^{(E/R)\cdot(1/T-1/413)} \quad \text{and } M_W=k_2\cdot([M]/[I_0])+k_3 \quad (2)$$

Correspondence: Dr. F. Couenne[a]
[a]Lab. Automatique et Génie des Procédés
UMR CNRS 5007, UCB Lyon 1/ESCPE Lyon
Université Claude Bernard Lyon1,bat 308 G
69622 villeurbanne CEDEX
Tel.: 33 4 72 43 18 82
Fax.: 33 4 72 43 16 82
E-Mail: couenne@lagep.univ-lyon1.fr
[b]Lab.des matériaux plastiques et des biomatériaux
UMR CNRS 5627, UCB Lyon 1/ISTIL
Université Claude Bernard Lyon1,bat 308 G
69622 villeurbanne CEDEX

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weight in weight in the limit Newtonian viscosity function.

The identification of dynamical parameters (not the delay) of the models is performed *via* the model output error equation method by using the Marquard-Levenberg optimization method.

To excite the process we have used a classical method, which consists of applying step changes to the input variable and subsequently measuring the open loop response of the output. There are other excitation methods more efficient such as pseudo-random binary sequence (PRBS) or relay feedback, but for experimental reasons, these methods could not be used.

For the first range of variation of the ratio $[M]/[I_0]$, the parameter of the model a_1 obtained after identification is equal to 41.3. By copying the pressure measured in experiments with the output of the model, we have observed that the model predicted perfectly the variations of the pressure. For the second model that corresponds to the second range, the value of the parameter a_2 is equal to 25.2.

3. Robust Control

The concept of robustness is related to uncertainties carrying either on the process itself, or on the environment of the process such as not modeled disturbances. Thus, when the model is not sufficiently precise, the use of the robust control proves to be useful because it enables some model errors to be tolerated.

3.1. *Uncertainty*

There are various ways in which an uncertain system can be modeled according to the form of selected uncertainties. In our case, we have chosen multiplicative uncertainties $|\Delta_m|$ [2]:

$$|\Delta_m| = |\tilde{P} - P|/|P| \qquad (5)$$

where P represents the uncertain model, given by:

$$P(s) = k \cdot e^{-\tau s}/(as+1)^2 \qquad (6)$$

with: $0.80 \le k \le 1.20$; $80 \le \tau \le 120$ and $9 \le a \le 57$ and $\tilde{P}$ represents the nominal model which is given by the average of the two models defined previously as follows:

$$\tilde{P} = e^{-100s}/(33s+1)^2 \qquad (7)$$

From the two models: the disturbed one and the nominal one, we plot the module of $|\Delta_m|$ according to the pulsation ω and we define W_2 as a boundary on the multiplicative uncertainties.

The transfer function of this bound is given by:

$$W_2(s) = (0.20 + 120s)/(1 + 6.66s) \qquad (8)$$

3.2. *The Control Law*

For the regulation of the pressure gradient we have chosen the internal model control (IMC). The diagram block of the IMC is represented in Fig. 1A [2].

$\tilde{P}$ is the uncertain model, P the nominal model and Q the controller.

According to this diagram block, we have:

$$y/d = 1 - \tilde{P}Q = S$$

and

$$y/r = \tilde{P}Q = T \qquad (9)$$

where S characterizes the influence of the disturbances on the output. and T the influence of the measurement noise on the output.

In order to obtain the best closed loop performance, the controller must verify:
1) Minimize the function S to decrease the influence of the disturbances.
2) Minimize the function T to decrease the influence of measurement noise.

Note that by choosing the corrector, $C = Q/(1 - \tilde{P}Q)$, the IMC can be seen as a simple feedback control (see Fig. 1B).

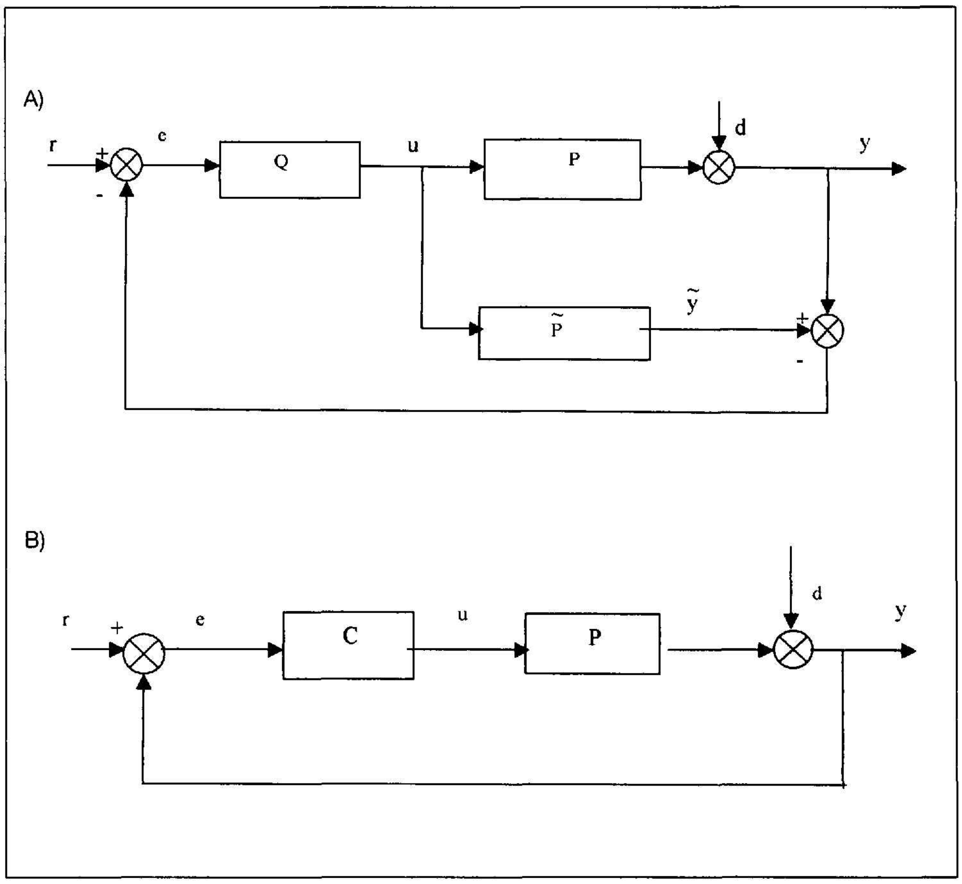

Fig. 1. A) IMC diagram block; B) diagram block of feedback control.

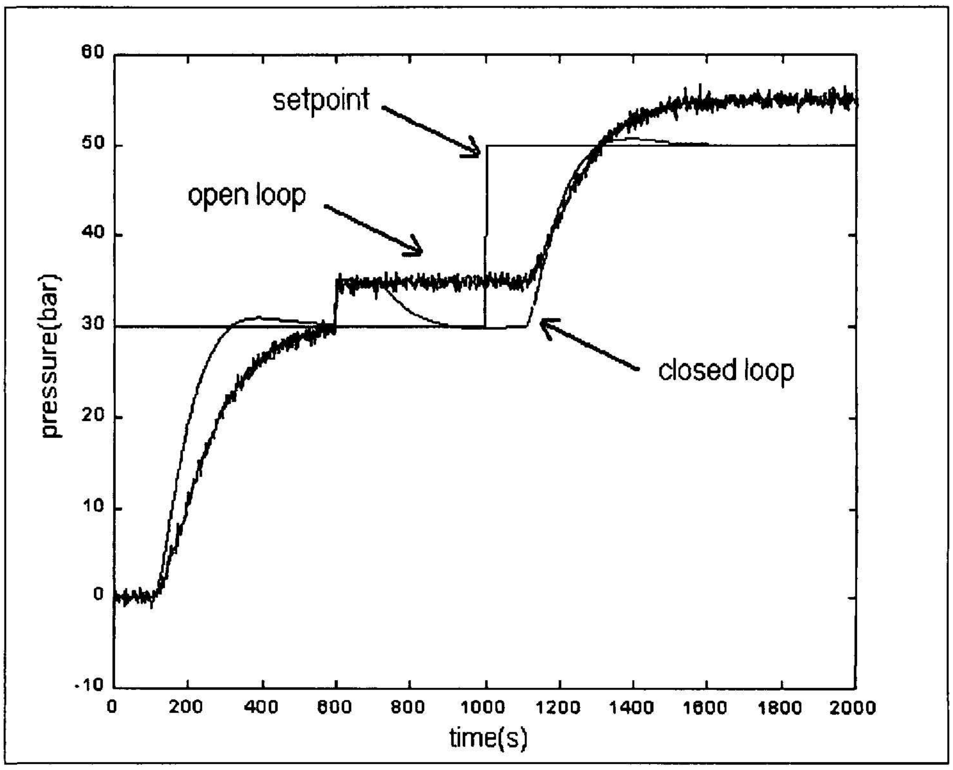

Fig. 2: Simulation results: pressure profiles.

For the synthesis of the controller, the nominal model is decomposed into a product of two factors (the first one regroups the unstable zeros and delays, the second one the minimum phase part of the transfer function):
$\tilde{P}=\tilde{P}_+\tilde{P}_-$, with $\tilde{P}_+=e^{-100s}$ and $\tilde{P}_-=1/(33s+1)^2$

The controller Q is chosen as follows:

$$Q=1/\tilde{P}_-\cdot f \Rightarrow Q=\tilde{Q}\cdot f \qquad (10)$$

f represents a low-pass filter (it is chosen such that Q can be realized physically) and its transfer function is given by:

$$f=1/(\lambda s+1)^2 \qquad (11)$$

The next stage to satisfy is the condition of robust stability [2]:

$$|T(j\omega)W_2(j\omega)|<1\ \forall\omega \qquad (12)$$

where $|W_2(j\omega)|$ is a boundary on the multiplicative error.

If this condition is satisfied then

$|\tilde{P}\tilde{Q}fW_2|<1\ \forall\omega$ (we vary λ until it is verified).

In the same way, robust performance [2] has to be performed. If it is achieved, the following condition is satisfied

$$|T(j\omega)W_2(j\omega)|+|S(j\omega)+W_1(j\omega)|<1\ \forall\omega \qquad (13)$$

$$\text{with: } |S(j\omega)|<|1/W_1(j\omega)| \qquad (14)$$

which implies:

$$\forall\omega\quad |TW_2|+|SW_1|<1 \Rightarrow |\tilde{P}\tilde{Q}fW_2|+|(1-\tilde{P}\tilde{Q}f)W_1|<1$$

(we vary λ until it is verified).

When λ is obtained, it is replaced in f and Q in C in order to have classic feedback control. In our case, for λ = 49 robust stability and robust performance are satisfied.

Fig. 2 shows simulation results for a pressure gradient setpoint of 30 bars followed by 50 bars. For this simulation, a disturbance step signal (of magnitude 5) is added to the measure at t = 600s and we observe that the controller rejects this disturbance and follows the setpoint. On the other hand, this perturbation is accumulated if the process is in open loop.

The results of the simulation obtained with this controller are satisfactory. This controller can stabilize the closed loop system with acceptable performances on all the domain of uncertainty.

4. Conclusion

The simulation results show that this simple robust control gives a good performance with this type of process (two second order dead-time transfer functions). However, the presence of more than two models cannot be managed so easily. In this case, other methods of control will be more adapted such as the multi-models control. It should be also noted that the temperature is a parameter which has a strong influence on the process, it is thus interesting to introduce it during the development of the model.

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